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Medical mathematics notes

1. Calculations based on heart rate

For a person with a normal, healthy heartbeat, pulse rate and heart rate is essentially the same thing. This is because every contraction of the heart produces a temporary increase in pressure (the "pulse") in the arteries [1].

Normal pulse rates at rest (bpm) [2]:

newborn (0–3 months old)	**infants (3 – 6 months)**	**infants (6 – 12 months)**	**children (1 – 10 years)**	**children over 10 years & adults, including seniors**	**well-trained adult athletes**
100-150	90–120	80-120	70–130	60–100	40–60

$VO_2max = 15 \cdot \frac{HR_{max}}{HR_{rest}}$ [3]

VO_2max: Maximum rate of oxygen consumption $\left[\frac{ml}{min \cdot kg}\right]$

HR_{max}: Maximum heart rate $[bpm]$

HR_{rest}: Resting heart rate $[bpm]$

$HR_{max} = 220 - age[a]\ [bpm]$ [4]

$$EE\left[\frac{kcal}{min}\right] = -59.3954 + gender \cdot \left(-36.3781 + 0.271 \cdot age[a] + 0.394 \cdot weight[kg] + 0.404 \cdot VO_2max\left[\frac{ml}{min \cdot kg}\right] + 0.634 \cdot HR[bpm]\right) + (1 - gender) \cdot \left(0.274 \cdot age[a] + 0.103 \cdot weight[kg] + 0.380 \cdot VO_2max\left[\frac{ml}{min \cdot kg}\right] + 0.450 \cdot HR[bpm]\right)$$

where $gender = 1$ for male and $gender = 0$ for female [5].

EE: Energy expenditure

A new study suggests that a higher resting heart rate is an independent predictor of mortality — even in healthy people in good physical condition [6].

Heart rate at rest (bpm)	≤ 50	51-70	71-80	81-90	>90
Long lifespan probability	very high	high	medium	low	very low

2. Body temperature

The temperature is measured on the forehead. The forehead temperature is usually 0.3°C to 0.6°C lower than the oral temperature [7]. Therefore

$$t_{oral} = t_{forehead} + 0.45°C \pm 0.15°C$$

Temperature classification [8]:

Temperature	<35.0 °C	36.5–37.5 °C	>37.5–38.3 °C	>37.5–38.3 °C	>40.0–41.5 °C
Diagnosis	Hypothermia	Normal	Fever	Hyperthermia	Hyperpyrexia

We use naïve Bayes classifier to estimate if a person is healthy or not based on the symptoms sneeze, cough and fever.

Bayes rule:

$$P(c_k|\{A\}) = \frac{P(\{A\}|c_k) \cdot P(c_k)}{P(\{A\})}$$

Denominator is evidence: probability of observations. If priors $P(c_k)$ and likelihoods $P(\{A\}|c_k)$ are known, the most probable class can be determined without computing evidence $P(\{A\})$.
If we assume each data attribute (symptom) independent given the diagnosis (class), then

$$P(\{A\}|c_k) = \prod_{i=1}^{N} P(A_i = v_j|c_k)$$

Example [9]:

Symptoms: sneeze, cough, fever

Diagnosis: allergy, cold, healthy

Prob.	**Allergy**	**Cold**	**Healthy**
$P(C_i)$	0.05	0.05	0.9
$P(sneeze \mid C_i)$	0.9	0.9	0.1
$P(cough \mid C_i)$	0.7	0.8	0.1
$P(fever \mid C_i)$	0.4	0.7	0.01

Target patient:

Sneeze	Cough	Fever
True	True	False

$$P(allergy|E) = \frac{0.05 \cdot 0.9 \cdot 0.7 \cdot (1 - 0.4)}{P(E)} = 0.0189$$

$$P(cold|E) = \frac{0.05 \cdot 0.9 \cdot 0.8 \cdot (1 - 0.7)}{P(E)} = 0.0108$$

$$P(healthy|E) = \frac{0.9 \cdot 0.1 \cdot 0.1 \cdot (1 - 0.01)}{P(E)} = 0.00891$$

As $0.0189 > 0.0108 > 0.00891$, the patient should be affected with allergy.

References

[1] http://www.diffen.com/difference/Heart_Rate_vs_Pulse

[2] http://en.wikipedia.org/wiki/Pulse

[3] http://en.wikipedia.org/wiki/VO2_max

[4] http://en.wikipedia.org/wiki/Heart_rate#Maximum_heart_rate

[5] http://www.braydenwm.com/cal_vs_hr_ref_paper.pdf

[6] http://well.blogs.nytimes.com/2013/04/19/heart-rate-as-a-measure-of-life-span/?_php=true&_type=blogs&_r=0

[7] https://myhealth.alberta.ca/health/pages/conditions.aspx?hwid=tw9223

[8] http://en.wikipedia.org/wiki/Human_body_temperature#Methods_of_measurement

[9] Marco Bernardo, *Formal Methods for Eternal Networked Software Systems*