The Art and Science of Selecting Robot Motors

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Selecting drive motors for a mobile robot is no simple matter. One may do it by guess, trial and error, or someone's advice. Having relied on all these methods in the past, with mixed success, I set out to find a better way. Selecting one of the many small gearhead motors that are widely available may be sufficient for small robots. For larger robots or those with challenging tasks, the methodology presented below will give more reliable results. While this overall approach may not be unique, I developed it to be practical and accessible for amateurs and hobbyists. Others may find it useful as well. Among the many kinds of electric motors that exist, I will restrict the discussion to brushed, permanent magnet DC (PMDC) motors (brushless PMDC motors are not considered). These motors are frequently used to drive robots weighing from a few to a few hundred pounds. While the motor sizing method presented here is illustrated with a ground-based, wheeled robot, the technique is applicable to any device for which the torque load can be estimated.

Although I have chosen a robust platform to illustrate the method, it is applicable to smaller, as well as larger, robots. The robot chosen for the analysis is the QuadRover manufactured by Parallax, Inc. (no longer commercially available). The original QuadRover used a gasoline engine connected to a hydraulic pump to power hydraulic motors, Photo 1.

Photo 1. Parallax QuadRover

The gas engine and hydraulic system was removed and replaced with PMDC motors. Other modifications include a new roll cage, a chain drive, lead acid batteries, Sabertooth 2x60 motor driver, Arduino Mega computer, and interface electronics, Photo 2.

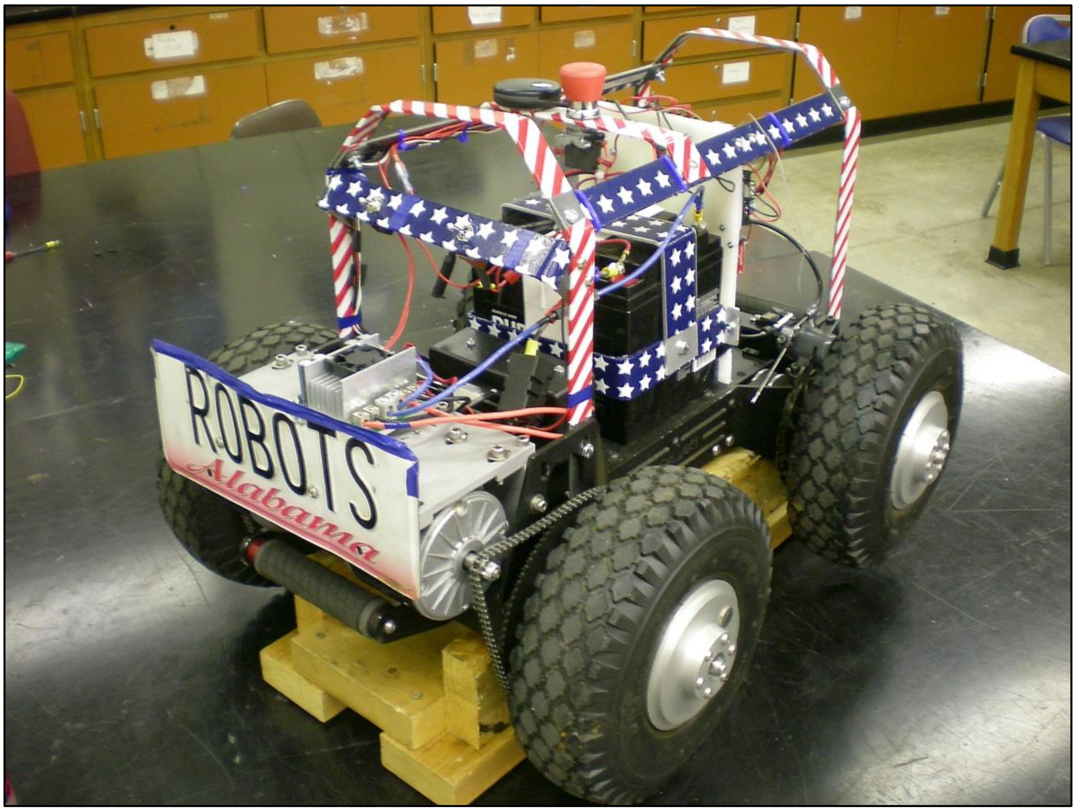

Photo 2. Green Rover (QuadRover Electric Conversion)

We will first examine the characteristics of PMDC motors, then develop motor performance requirements for mobile robots moving at more or less constant speed, which is usually sufficient for most robot applications, compare the motor predicted and measured performance and then expand the methodology to the more complex case of accelerated motion. Finally, we will determine the battery requirements to power the motors.

I. PMDC Motor Characteristics

We begin by getting familiar with the basic characteristics of PMDC motors, which, fortunately, have a simple performance curve. The main characteristics of concern are the motor angular speed (spin rate), drive torque and electric current, from which we derive power and efficiency. Figure 1 illustrates the interrelationships of these performance quantities.

The main performance curve of interest is the straight line that slopes down from the upper left to bottom right relating motor speed to output torque. Motor speed is usually expressed in revolutions per minute (rpm) or less frequently radians per second (rad/sec) and output torque in Newton-meters (Nm), foot-pounds (ft-lbs.) or ounce-inches (oz-in). On-line sites provide convenient conversions among the various units. A particularly useful site is: www.onlineconversion.com. The straight line sloping from the lower left to the upper right is the electrical current the motor draws, which depends on the load the motor is driving. Vendors of used motors don't usually supply the full performance curve but rather only give two points on the curve. These descriptions may or may not be sufficient to analyze the motor performance. The minimum information necessary to fully characterize a PMDC motor are two points on the speed vs. torque and two points on the current vs. torque lines. These values may be:

- The no load speed, ω_o; the speed at zero torque, when the motor is freely spinning with no external load.
- The no load current i_o.
- The stall torque, Ts; the torque when the load on the motor just prevents it from turning.
- The stall current i_s.

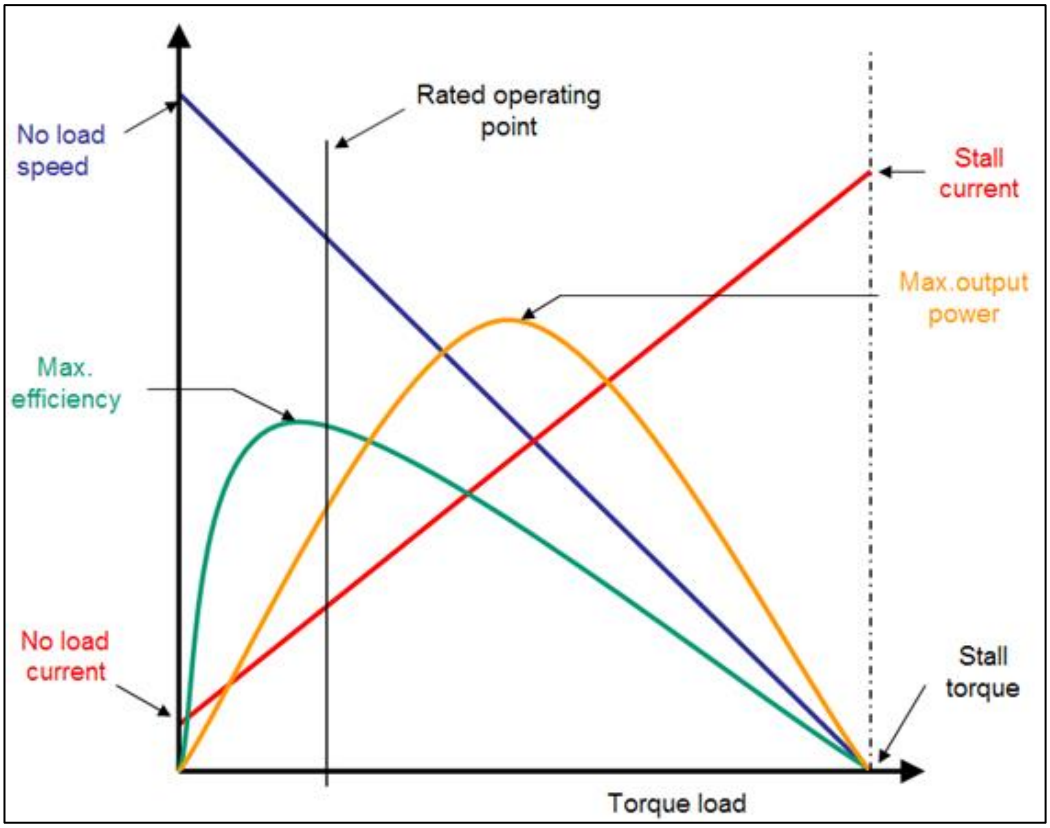

Figure 1. Generic Motor Performance Curves

Instead of the stall values, the usual practice is to specify a rated operating point. Since the speed and current vs. torque relationships are straight lines, one can find the stall values from the rated operating point values by extrapolation (see equations 1, 2, 3 or 4 below). The rated operating point is important because it is usually the maximum torque and current the motor can operate at continuously without overheating. This point specifies the power rating of the motor.

The other curves on the graph are the output power, the curve with its maximum at half of the stall torque, and the motor efficiency. It's important to note that motors have a maximum current, set by the load torque, at which they can operate continuously. This current is sometimes given indirectly as the motor's rated power (which is the *mechanical* output power). If that value is exceeded for all but a short period of time, by attempting to drive too heavy a load, the motor will overheat and then burn out. This maximum continuous power value is usually located a little past the maximum efficiency value. For many motors, operating at no more than 15% of the stall torque is safe.

<u>Motor Performance Curves</u>

The equation of the straight line for the speed vs. torque may be written as:

$$\boldsymbol{\omega = \omega_o\,(1 - T/Ts)} \qquad \textbf{equation 1}$$

or

$$\mathbf{T = Ts\,(1 - \omega/\,\omega_o)} \qquad \textbf{equation 2}$$

The equation for the current is:

$$\mathbf{i = i_o + (i_s - i_o)} \times \mathbf{T / Ts} \qquad \textbf{equation 3}$$

or

$$\mathbf{i = i_o + (i_S - i_o) \cdot (1 - \omega/\,\omega_o)} \qquad \textbf{equation 4}$$

From equation 3, we can express the torque as a function of current:

$$\mathbf{T = Ts\,(i - i_o) / (i_s - i_o)} \qquad \textbf{equation 5}$$

A motor's mechanical output power is the product of its speed and corresponding torque:

$$\mathbf{P}\ \text{(watts)} = \boldsymbol{\omega}\ \text{(rad/sec)} \cdot \mathbf{T}\ \text{(Nm)}$$

Note that all calculations must be performed in consistent units. If employing other units, they must be converted to units that are consistent with the rest of those in the equation. For instance, in the English system of units,

$$\mathbf{P}\ \text{(watts)} = \mathbf{0.738}\ \boldsymbol{\omega}\ \text{(rad/sec)} \cdot \mathbf{T}\ \text{(ft-lb)} = \boldsymbol{\omega}\ \text{(rpm)} \cdot \mathbf{T}\ \text{(ft-lb)}\mathbf{/7.05} \qquad \textbf{equation 6}$$

Multiplying $\boldsymbol{\omega}$ in eq. 1 by $\mathbf{T}$ gives,

$$\mathbf{P = \omega_o\,T - (\omega_o/\,T_S) \cdot T^2.}$$

Thus we see that the motor output power curve is a parabola with a value of zero when T equals zero or Ts, and has a maximum of ($\boldsymbol{\omega_o}\ \mathbf{T_S/4}$) Watts at a torque of Ts/2.

The motor efficiency is the ratio of mechanical power produced divided by the electrical power consumed.

$$\epsilon = \frac{\mathbf{T}\ \left(\omega/7.05\right)}{\mathbf{V_o} \cdot \mathbf{i}}$$

$$\epsilon = \frac{\mathbf{T_S}\left(\mathbf{1} - \frac{\boldsymbol{\omega}}{\boldsymbol{\omega_o}}\right)\boldsymbol{\omega}}{\mathbf{7.05} \cdot \mathbf{V_o}\left[\mathbf{i_o} + (\mathbf{i_S} - \mathbf{i_o})\left(\mathbf{1} - \frac{\boldsymbol{\omega}}{\boldsymbol{\omega_o}}\right)\right]}$$

where the units are:
Ts – stall torque (ft-lb)
ω – motor speed (rpm) = 229·v/D, v is the wheel speed (ft/sec) and D is wheel diameter (inches)
ω_o – no load speed (rpm)
i_o – no load current (amps)
i_S – stall current (amps)
V_o – nominal operating voltage (volts)
7.05 is a constant to make the units consistent.

The maximum efficiency is approximately :

$$\varepsilon_{max} = \left(1 - \sqrt{\frac{i_o}{i_S}}\right)^2 \qquad \textbf{equation 7}$$

and the speed at which the efficiency peaks is approximately:

$$\omega_{max} = ½(\omega_o + 7.05 \cdot \varepsilon_{max} \cdot i_S \cdot V_o/T_S)$$

The waste power generated is the electrical power consumed minus the mechanical power produced, or the inefficiency times the power consumed.

$$P_W = (1 - \varepsilon) \cdot V_o \cdot i$$

The Figure 2 illustrates how rapidly waste heat is generated at higher torques.

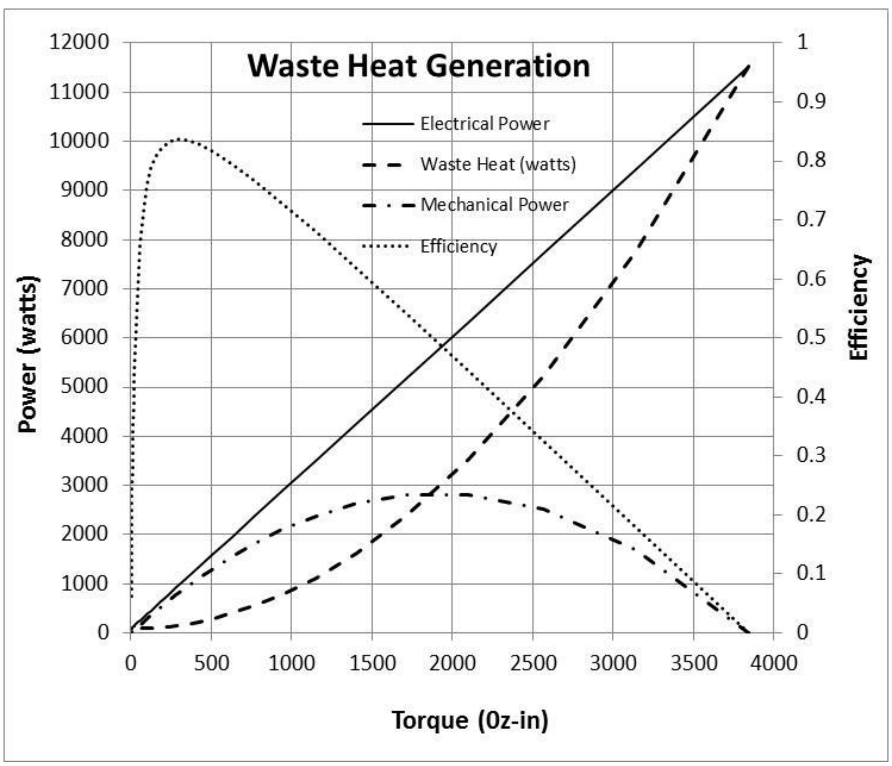

Figure 2. Waste Heat Generation

We have shown the motor performance curve when operating at the full, rated voltage, however, motors are not frequently operated at that value. Motor speeds are changed by changing the effective, applied voltage. This is usually done electronically through the use of a motor driver circuit. Doing so changes the speed vs. torque curve, shown in Figure 3, linearly with the ratio of the applied voltage, V, to the rated voltage, V_o. The speed vs. torque line also changes if gearing is added to the output of the motor. If the motor comes with a gear head, then the motor performance numbers already take the gearing into account. If gearing is added, then the performance curve needs to be modified. A gear ratio that reduces the output speed, increases the torque by the same amount, the gear ratio. For instance, a 5-to-1 speed reduction gear, will decrease the no load speed by a factor of five and increase the stall torque by a factor of five.

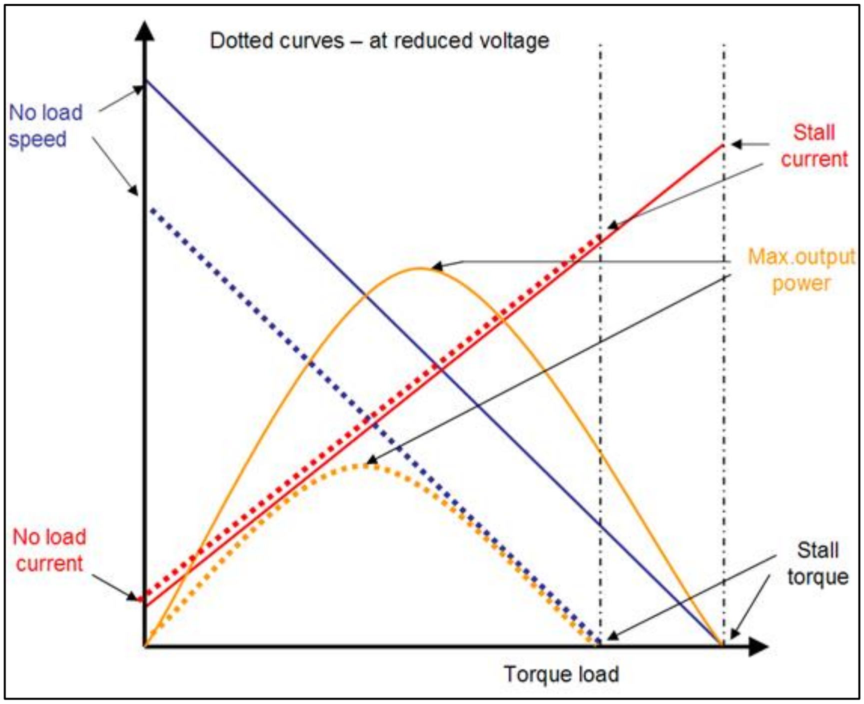

Figure 3. Motor Performance at Reduced Voltage

Without going into the theory of PMDC motors, Figure 3 shows that the motor speed is proportional to the voltage and the torque is proportional to the current. Motor manufacturer's usually provide a host of fundamental specifications. The most important are the speed constant, k_v, and the torque constant, k_t. The motor speed is then given by: $\omega = k_v * V$, and the torque by $T = k_t * i$.

The motor performance curves in Figures 1 – 3 all assume that the current is not limited by the power supply. When the current is limited, usually at high torque loads, the torque will also be limited, as shown in Figure 4. It may be difficult to determine if and when this condition occurs without current sensors. Most motor drivers are rated with a max. output current. Current limiting affects the motor performance calculations.

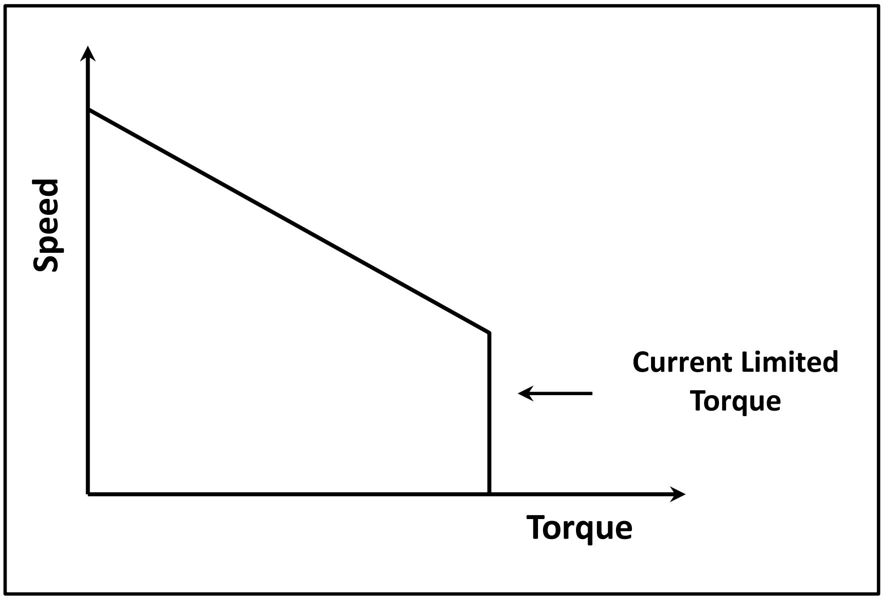

Figure 4. Current Limited Torque

Now that we have PMDC motor performance expressed graphically and algebraically, we move on to determining motor specifications from robot performance requirements.

II. Constant Speed Analysis

We proceed by developing a motor selection method for the QuadRover, starting with robot parameters and performance requirements, and determining the motor performance specifications. The analysis is used to specify the motor power and gear ratio.

Robot Parameters – the modified QuadRover

- 90 lb. mobile platform
- Two wheel motor drive
- Drive motors attached to the wheels via a chain drive and sprockets
- Skid steering
- 10.6 inch diameter rubber wheels
- Vehicle aerodynamics, $C_DA = 1.6\ ft^2$ (see discussion below)

Performance Requirements – desired capabilities

- Achieve a peak speed of 15 mph (22 ft/sec)
- Ascend inclines from 0 to 15 degrees (27% grade)
- Drive over a grassy surface

Load Torques

For a constant robot speed, the motor torque load is determined by three physical factors: rolling resistance, ground slope, and air drag. We will lump drive train inefficiencies in with rolling resistance.

1. rolling resistance: Compared to static friction (which occurs before motion starts) or dynamic friction (when an object slides across a surface), rolling friction (usually called rolling resistance) is very small. The basic equation for the resistive force due to rolling resistance is:

$$\mathbf{F = C_{rr} \cdot W}$$

where
F is the force in lbs.
W is the weight of the robot in lbs.
$\mathbf{C_{rr}}$ is the coefficient of rolling resistance.

There are many parameters that affect rolling friction, including the robot internal mechanical friction and the interaction of the wheels with the surface. An estimate of rolling resistance can be made by pulling the robot horizontally across the surface of interest at constant speed. A spring or digital scale should give the approximate force. If a measurement is not practical, one may find tables of basic $\mathbf{C_{rr}}$ for a combinations of materials on the Internet.

2. ground slope: Driving a robot up a slope requires that the motor supply a force to counter the force of gravity pulling the robot down the slope. This depends on the steepness of the slope, which we characterize as the angle of the surface with the horizontal, theta **(θ)**. The down slope force is given by:

$$\mathbf{F = W \cdot \sin(\theta)}$$

where **W** is the robot weight in lb.

The weight of the robot directed perpendicular to the ground is $F = W \cdot \cos(\boldsymbol{\theta})$, which technically should be used as the weight in the equation for rolling resistance, however, we'll use the full weight so as to not underestimate the rolling resistance force.

3. air drag: The amount of air drag is a function of the robot's geometrical shape, size and speed. The quantities of interest are the drag coefficient, C_D , the frontal area, A, and the speed, v. An estimate of the drag force can be made knowing something about the cross sectional size and shape of the front of the robot, that is its projected area. Extensive look up tables for a variety of shapes are available on the Internet. Although we may ignore air drag for all speeds likely of interest to amateurs, we nevertheless include it here for those occasions where it becomes important.

The force required to move an object through a resisting medium, is given by:

$$\mathbf{F_D = C_D \cdot A \cdot \rho \cdot v^2/2}$$

where,
F_D is the drag force
C_D is the drag coefficient
A is the object's effective cross sectional area
ρ is the density of the medium
v is the speed

Physically, the drag force is the drag coefficient times the work done per unit distance in pushing the medium aside. Using 0.00237 slug/ft^3 as the density of air:

$$\mathbf{F_D = C_D \cdot A \cdot v^2/840}$$

where
F_D is the force in lb
C_D is the drag coefficient
A is the frontal cross sectional area in ft^2
v is the speed in ft/sec

Example Torque Calculation

Let's evaluate the torques associated with these forces for the robot parameters and performance requirements given above. The torque the motor must supply is just the wheel radius times the force it exerts on the surface it rests on. Since it is convenient to use the wheel diameter in inches, we calculate torque from:

$$\mathbf{T = F \cdot D/24,\ ft\text{-}lb}$$

where F is in lb. and D is in inches.

The forces and corresponding torques for our robot are:

1. Rolling resistance (using an estimated value of **C_{rr} = 0.08** for rubber on grass)

$$\mathbf{F = W \cdot C_{rr} = 0.08*90 = 7.2\ lb,\quad T = 3.2\ ft\text{-}lb}$$

This value was roughly verified by pulling the rover over a level, grass surface and noting the force on a digital scale.

2. A 15 deg. slope,

$$\mathbf{F = W \cdot \sin(\theta) = 90 \cdot \sin(15^o) = 23\ lb,\ \ T = 10.3\ ft\text{-}lb}$$

3. A speed of 15 mph (22 ft/sec),
The frontal area of the modified QuadRover is roughly 1.5 ft^2 and using a C_D for a cube of 1.05,

$$\mathbf{F = C_D \cdot A \cdot v^2/840 = (1.05) \cdot 1.5 \cdot (22)^2\ /840 = 0.9\ lb,\ \ T = 0.4\ ft\text{-}lb}$$

Thus for our performance requirements, **a total torque of T = 14 ft-lb** is required for constant speed up a 15 deg slope. Note that for **level ground only 3.6 ft-lb** is required, almost independent of speed. In fact, rolling resistance depends on more than the surfaces in contact. For pneumatic tires, there is also a dependence on inflation pressure, tire width, rubber compound and speed, since a large part of the resistance comes from tire deformation as it rolls.

Note that here, and elsewhere, we make approximations and round the result of calculations. This is justified due to various unknowns that are not modeled and we compensate by choosing motor and battery requirements in excess of those calculated.

Motor Speed

Torque is only part of the motor requirement; now we consider motor speed. Rotational speed is related to linear speed (no wheel slippage) by

$$\boldsymbol{\omega\ (rpm) = 229 \cdot v / D}$$

where v is in ft/sec and D in inches.

Then a speed of 15 mph (22 ft/s) with a 10.6 inch diameter wheel requires a motor speed of **ω = 475 rpm**.

So our constant speed motor requirement for hill climbing is a torque of 14 ft-lb and a motor speed of 475 rpm. From equation 6, this is a power rating of 943 Watts, or about **470 W per motor.** Thus we would need **two 500W** motors. However, since we don't plan to climb steep hills often or continuously, it may be more practical to size the motor for cruising on level ground and then evaluate the motor for short periods of hill climbing. For level ground the torque needed is 3.6 ft-lb. Then the power rating is 242W or **120W per motor**. Given that hill climbing takes almost four times as much torque as rolling over level ground, a couple of 120W motors will have very anemic hill climbing ability. We'll compromise at **350W per motor** (partly arrived at by iteration of the analysis given below - the "art" part of the process – and available motors).

Motor Requirements Summary

1. Motor torque
   - Level ground – 3.6 ft-lb
   - Hill climbing – 14 ft-lb
2. Speed – 475 rpm

Motor Specification

Based on the above requirements analysis and our compromise power rating, let's consider a commercial motor rated for continuous operation at 350 W. The performance parameters and performance curves furnished by the vendor are given below.

AmpFlow Model	**P40-350**
Watts – Continuous	350
Diameter (inches)	4.0
Length (inches)	4.3
Max Efficiency	79%
Voltage (volts)	24
No-Load RPM @24V	3500
Output Shaft	Sprocket
Pounds	5.6

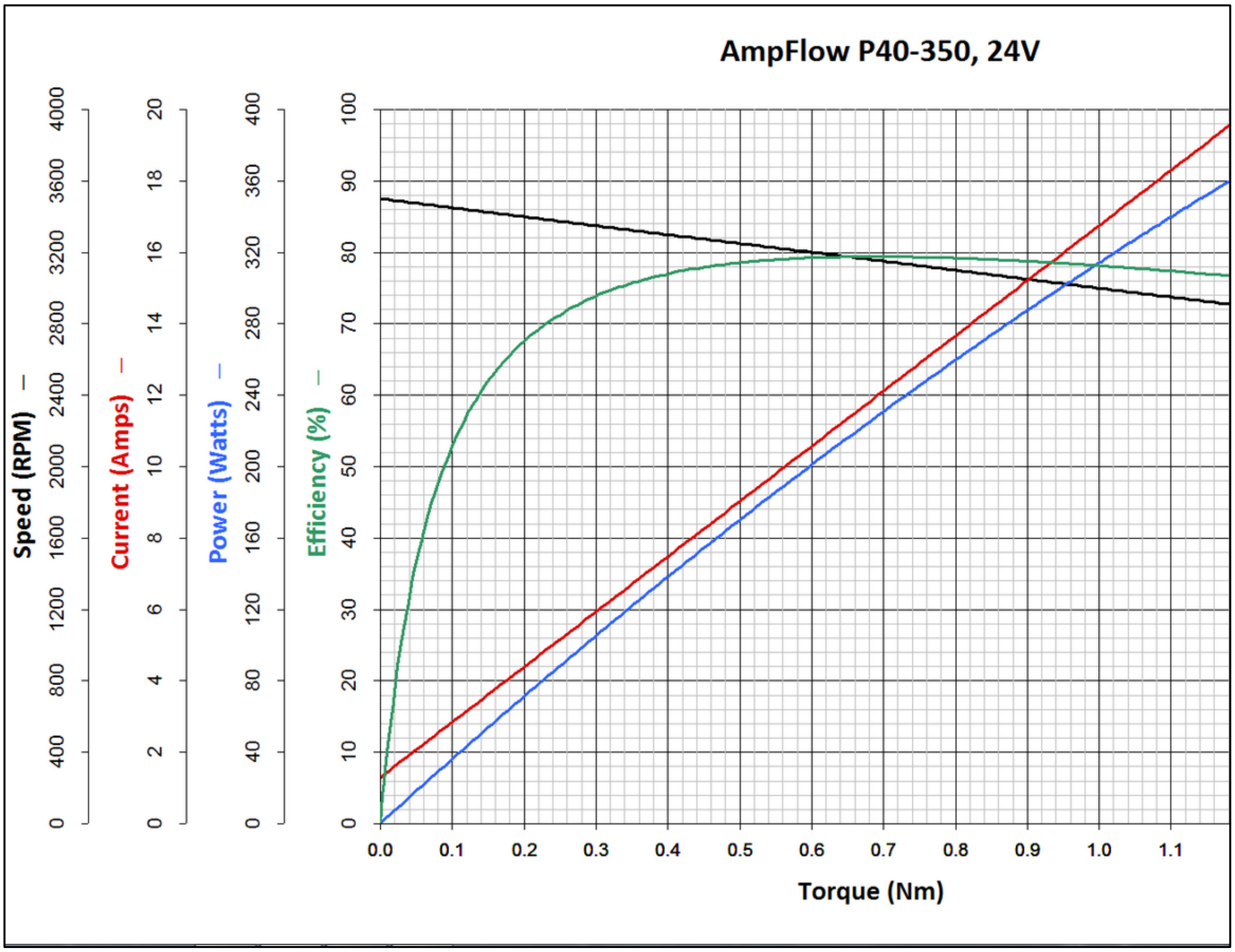

Figure 5. Motor Performance Curves

Note that the entire performance curve (Figure 5) is not shown. In this case, which is useful but not typical, only the portion rated for continuous operation is shown. This is an important reminder which most motor specifications do not make. The power rating is for mechanical output power.

To complete the motor performance curve, let's calculate the stall torque and current of a single motor. Using a point at the max. continuous torque (any point on the graph will do) of 1.18 Nm (0.870 ft-lb), the corresponding speed of 2900 RPM and the no load speed of 3500 RPM, we calculate, from eq. 2:

$$\mathbf{T_S = 5.08 \text{ ft-lb}}$$

Using equation 4, with a no load current of 1.3 amps and a current of 19.6 amps at 1.18 Nm, we calculate a stall current of 112 amps. At its rated power then, the electrical power input is 470W.

<u>Motor Performance Summary</u>

1. No load speed – 3500 rpm
2. Stall torque – 5.08 ft-lb
3. No load current – 1.3 amps

4. Stall current – 112 amps
5. Rated mechanical output – 360W (note from Figure 5 this is larger than the nominal motor rating of 350W).
6. Rated electrical input – 470W
7. Efficiency at max continuous power – 76%

The first thing we notice in Figure 5 is that the speeds are considerably greater than the 475 rpm and the torques considerably less than 14 ft-lb that we need. Therefore an external gear is needed to transform the motor's intrinsic speed vs. torque curve into one that meets our motor performance requirement. *What gear ratio is needed to bring this motor closer to our requirements?*

Level Ground Analysis

Let's attempt to operate around the maximum efficiency point, which for the P40-350 motor occurs around 3200 rpm and 0.6 Nm (0.443 ft-lb). Dividing 3200 by 475 gives a ratio of about 6.7. The P40-350 motors come with an 11 tooth output sprocket, which is changeable. A gear ratio of 6.7 then requires a sprocket on the drive shaft with 74 teeth. At the time of purchase, there was a 6 week delivery time for 75 tooth sprockets (74 teeth not manufactured), therefore a sprocket with 65 teeth, a gear ratio of 5.9, was ordered. 5.9 times 0.443 ft-lb gives a torque of 2.6 ft-lb. Since we are using two motors, the total torque available is about 5.2 ft-lb, more than the 3.6 ft-lb needed for moving over level ground. A motor stall torque of 5.08 ft-lb with a gear ratio of 5.9 gives a **geared stall torque of 30 ft-lb and a geared, no load speed of 593 RPM**. From equation 1, the single motor torque of 1.8 ft-lb results in a speed of 557 rpm or about 18 mph. At this point, the motor current is 8 amps, the power consumed is 192 W and the mechanical power supplied is 142W, an efficiency of 74%. Due to some approximate calculations, the operating point is short ward of the maximum efficiency point and the speed faster than the objective of 15 mph.

Hill Climbing Analysis

The maximum available torque for continuous operation, from the performance curve, is 1.18 Nm, or for geared operation 5.9*1.18 N-m = 7.7 N-m or 5.7 ft-lb per motor. Not enough for hill climbing, so we must operate beyond the max power rating. Operating at an increased torque to ascend a hill will slow down the robot. However ascending steep slopes is usually only done for short times, thus we may be able to briefly operate the motor past its maximum continuous rating without a problem. *How fast will the robot be able to ascend a 15 deg. slope with two of these motors*? We only need 14 ft-lb, or 7 ft-lb for each motor. Using the equation 2, for torque vs. speed, we expect a torque of 7 ft-lb to occur at a speed of 454 rpm, a linear speed of 21 ft/sec or 14.5 mph. So far, so good, however, we are now asking the motor to put out 451W, which is beyond its 350W rating.

How much excess heating can we expect? At its rated power, the electrical power input is 470W. At a speed of 454 RPM, each motor will be using 27 amps. At 24 v. the electrical power consumed by each motor is then 654 Watts, the excess over 470W electrical power of 184 Watts supplying excess heat. How long a 350W motor can supply 451W depends on its construction and ability to dissipate heat. If of concern, either a higher rated motor or less steep slopes should be specified.

<u>Uniform Speed Summary – Gear Ratio 5.9</u>

Motor stall torque: 30 ft-lb
No load speed: 593 rpm.

Level Ground:

Gear ratio = 5.9
Torque Required = 1.8 ft-lb per motor
Motor speed = 557 rpm
Robot Speed = 18 mph
Current = 8 amps
Mechanical Power = 142 Watts
Electrical Power = 192 Watts

Uphill Sloped Surface (15 deg.) :

Gear ratio = 5.9
Torque Required = 7 ft-lb per motor
Motor speed = 454 rpm
Robot Speed = 14.5 mph
Current = 27 amps
Mechanical Power = 451 Watts
Electrical Power = 654 Watts

<u>Skid Steer</u>

For the QuadRover there is an additional issue not common to most wheeled robots, skid steering. The QuadRover is 4-wheel drive (each rear wheel is connected to the corresponding, driven front wheel with a belt). This gives excellent traction along with a large penalty in the power required for turning (the QuadRover may also be turned by braking one side). Skid steering takes more torque than other modes of operation except moving up inclines steeper than that we have considered. For our robot, the best value of the coefficient of sliding friction of rubber on grass I could find is 0.35. During skid steering two tires on the same side of the vehicle move in one direction, two on the other side in the opposite direction and all four skid sideways. With a friction coefficient of 0.35, the force required to skid rotate the 90 lb robot is about 31.5 lb. The torque lever arm for sideways skidding is the right angle distance between the tire axle and the center of the platform, about 13 in., therefore the torque required is about 34 ft-lbs, or 17 ft-lbs per motor, greater than our previous torque estimates. This can only be a rough estimate. An experimental determination, made using a couple of spring scales pulling diagonally on opposed wheels, gave a value of 28 ft-lb (14 ft-lb per motor), surprisingly good agreement with the predicted value of 31.5 lb.

The equations developed in Section I are not applicable for calculating the current required for skid steering since the torque supplied by the motor is perpendicular to the torque causing skidding. The motion is complicated because the tires are turning as well as skidding. A rough estimate of the current using the measured value of 14 ft-lb per motor, from equation 3, is 53 amps.

Skid Steer:

Gear ratio = 5.9

Torque Required = measured: 14 ft-lb per motor; sliding friction: 17 ft-lb

Current = 53 amps from measured torque; 64 amps from coefficient of sliding friction

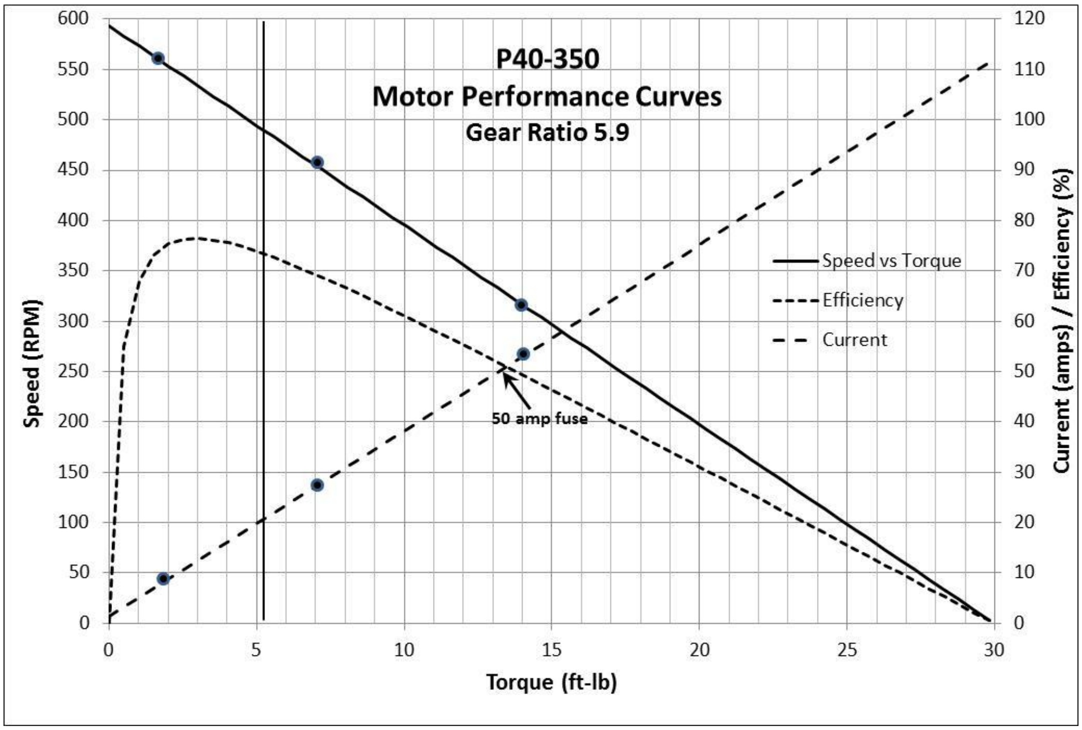

Figure 6. Motor Performance: Gear Ratio 5.9

Figure 6 shows the motor performance curves with a 5.9 gear ratio. The vertical line marks the motor rating of 350W and the dots show the speed, torque and current for (left to right) level ground cruising, hill climbing and skid steering. For this gear ratio, the skid steering power not only is far beyond the motor's rating, but for some surfaces, blew the 35 amp motor fuses.

To overcome the skid steering problem, the gear ratio was changed to 8.6 (95 tooth sprocket) and the fuses upgraded to 50 amps. The re-geared motor performance curve is shown in Figure 7. Now hill climbing is within the motor rating and the skid steering current is within the limit of the time delay 50A motor fuses. The **revised** uniform speed summary is:

<u>Uniform Speed Summary – Gear Ratio 8.6</u>

Motor stall torque: 44 ft-lb

No load speed: 407 rpm (18.8 ft/sec, 12.3 mph).

Level Ground:

Gear ratio = 8.6

Torque Required = 1.8 ft-lb per motor

Motor speed = 390 rpm

Robot Speed = 12 mph

Current = 5.8 amps

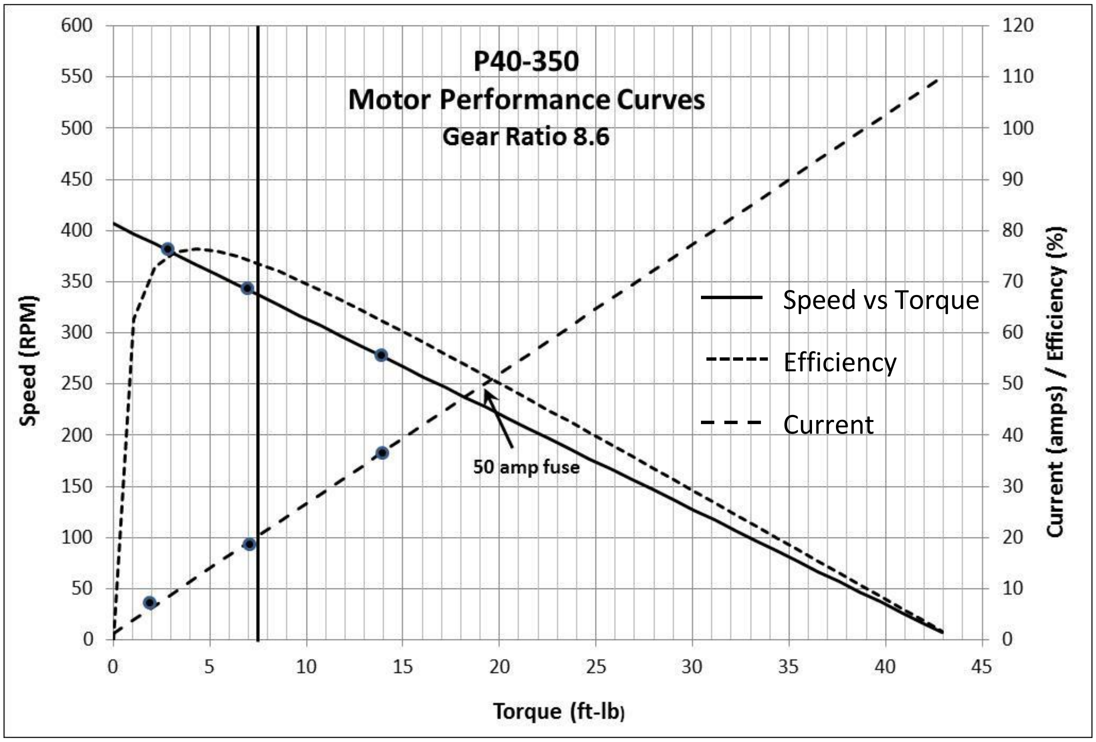

Figure 7. Motor Performance Curve: Gear Ratio 8.6

Mechanical Power = 100 Watts
Electrical Power = 140 Watts
Efficiency = 71% (operates short ward of max efficiency)

Uphill Sloped Surface (15 deg.) :

Gear ratio = 8.6
Torque Required = 7 ft-lb per motor
Motor speed = 342 rpm
Robot Speed = 11 mph
Current = 19 amps
Mechanical Power = 340 Watts
Electrical Power = 454 Watts
Efficiency = 75%

Skid Steer:

Gear ratio = 8.6
Torque Required = 14 - 17 ft-lb per motor
Current = 36 amps
Electrical Power = 865W
Efficiency = 57%

Note that in order to accommodate the skid steer motor currents, we have changed gear ratio and the original target speed of 15 mph is no longer attainable. Instead the max speed we can hope to achieve now is 12 mph. Since the 15mph value was arbitrary to begin with, this isn't a serious compromise.

III. Comparison with Measurements

How well does the motor analysis predictions compare to measurements?

Level Ground

Gear ratio = 8.6	
Robot Speed = 12 mph	Measured 12.3 mph
Current = 5.8 amps	Measured 5.8A on asphalt
	6.2A on grass

It's not surprising that the level ground speed prediction is very close to the measured values since the load torque running on level ground is so low that the motors are practically running full tilt. The higher than predicted current indicates the estimated torque, probably rolling resistance, is higher than estimated. We expect, from Figure 3, the speed to scale linearly with the applied voltage and this was confirmed (Figure 8) by comparing the measured speed with that predicted by scaling the speed by the ratio of the voltage across the motors to the battery voltage.

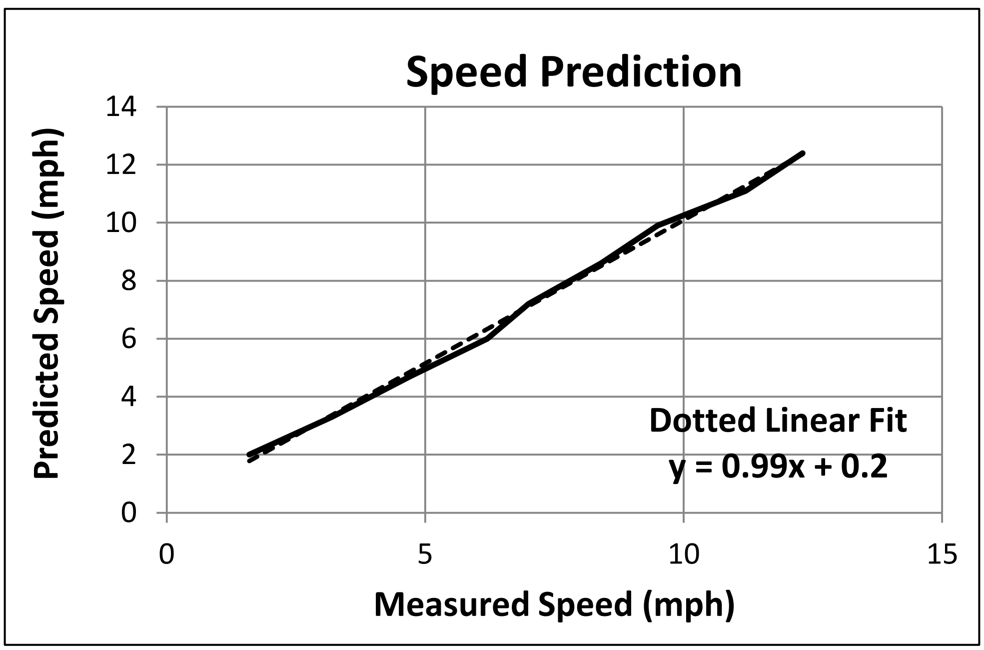

Figure 8. Comparison of Measured and Predicted Speeds

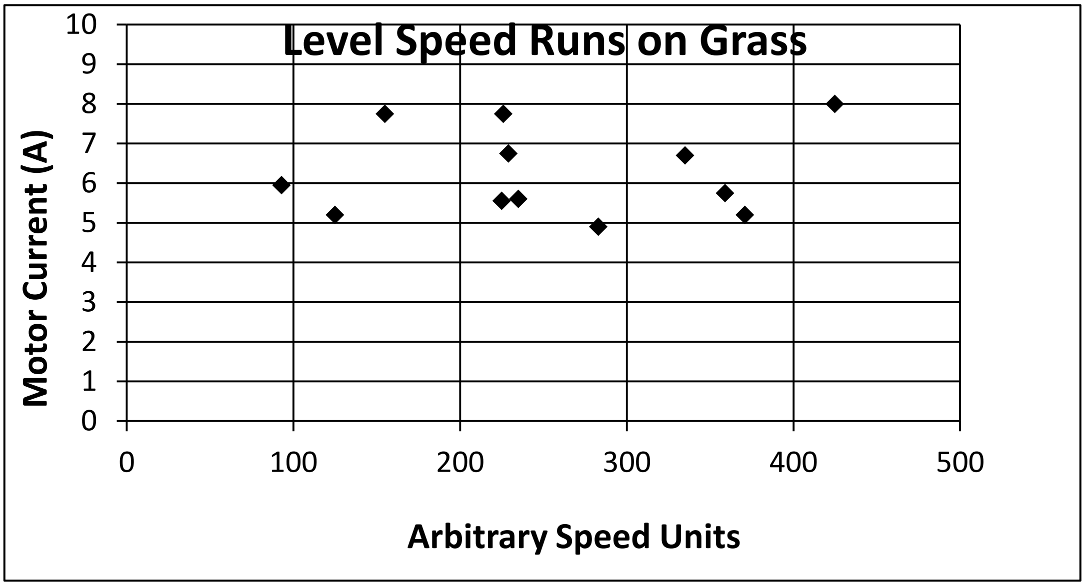

Figure 9. Rover Motor Current vs. Speed

The rover current is, with a lot of scatter, similar for different speeds, Figure 9. The current depends on the particular turf – kind of grass, height, bumpiness, etc. The measured motor current varies between 5 and 8 amps. The average is 6.2A. To the degree that all of our torques are speed independent, we expect the current to be similar at different speeds The x-axis indicates the speed difference from the stop position and varies from slow to full throttle.

A test of the validity of the slope calculations are current measurements on inclines. Due to the unavailability of long inclines of constant slope, the currents were taken while the rover ascended slopes of increasing inclination at low speed. The low speed avoids the effect of acceleration and bumps that effect the motor current and the inclinometer. Low speed currents for motion on level ground are then subtracted from the slope measurements to deduce the current required for ascending the incline only. The current is for both motors combined.

Uphill Sloped Surface (8.6 gear ratio)**:**

Slope (deg)	Measured Current (A)	Predicted Current (A)
0	0.0	0.0
5	9.5	10.0
10	19.1	18.6
15	28.6	27.2
22	40.4	39.4

The measurements were made over different slopes and averaged. Given the variation in the measurements due to uneven ground, the agreement is closer than expected. Roughly, twice the slope in degrees equals the current (both motors combined) in amps.
The skid steering current is also in the range of the rough estimates made from both digital scale measurements of the torque and value of the coefficient of sliding friction.

Measured Skid Steering Currents

1. sidewalk concrete – 40 amps
2. linoleum – 35 amps
3. asphalt – 30 amps
4. brick – 40 amps
5. grass – 40 amps
6. low nap carpet – 45 amps

Predicted Skid Steering Current

36 amps from measured torque
45 amps from sliding friction 0.35

A typical skid steering current plot is shown in Figure 10.

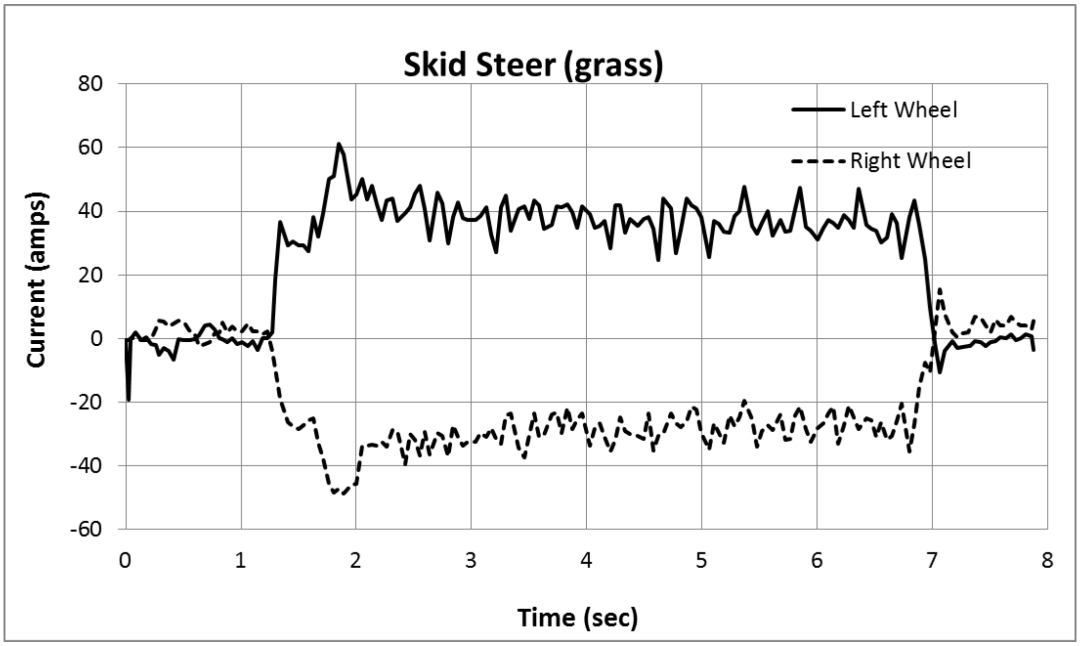

Figure 10. Skid Steer Motor Currents

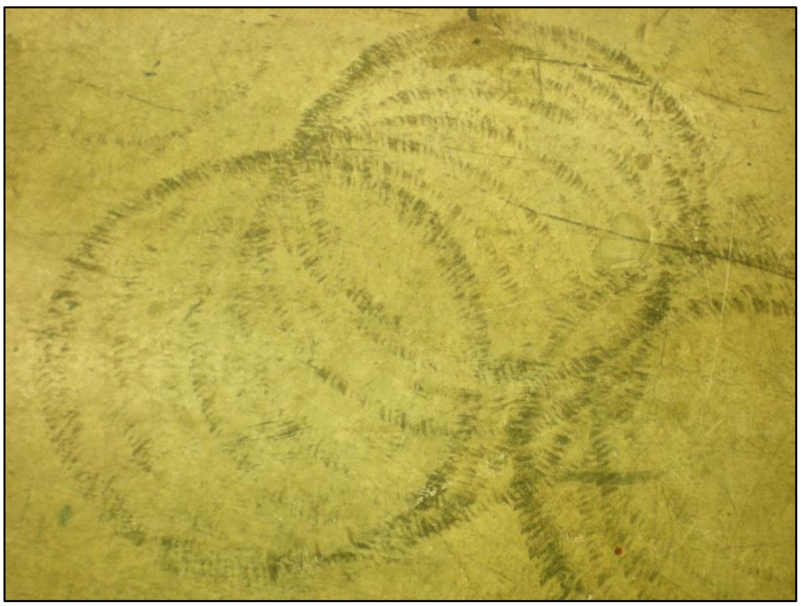

Photo 3. Skid Steering Tracks on Concrete Floor

The current is irregular in part due to the "catch and release" action of the skidding tires (Photo 3). The rover makes several revolutions in the space of a few seconds.

As we might expect from the nature of sliding friction, once the skid starts, the current is sensibly the same whether turning at a slow or fast rate, The initial current is higher for faster turn rates, as required to impart a higher angular momentum.

IV. <u>Accelerated Motion Analysis</u>

Acceleration normally only occurs for a brief period of time and is not usually a primary concern. As we shall see, the acceleration from zero to cruise speed is exponential. However, there are applications in which the acceleration phase is very demanding.

Two examples from my personal experience:

1. A student contest in which a ball striking head was carried back and forth across the width of the playing field on a rail. In order to intercept an incoming ball in time, the head had to move very rapidly from side-to-side.

2. A commercial company with a requirement to design and build a several hundred pound robot to accelerate from zero to 55 mph in ten seconds or less. In this case, air drag was important.

In both these cases, acceleration was key to success. For accelerated motion, the velocity is continuously changing and not necessarily at a uniform rate but at a rate determined by the robot and the environment. Therefore we must use time dependent equations of motion. These are involved and a derivation of the equations are given in the End Note. Here we present the equations and apply them to our example. When using the equations, it's probably a good idea to program them or put them in a spread sheet and let the computer do the computations. A cautionary note, *only linear acceleration is considered*. The torques required to spin up the wheels are neglected.

<u>Equations of Motion</u>

The motor speed vs. torque relationship is the force law governing the equations of accelerated motion. In addition to acceleration, we also include the effect of external forces due to slopes, air drag and rolling resistance. Then using Newton's second law, we can derive the equations of motion.

The equations give:

1. Time to accelerate to a given speed
2. Speed vs. time
3. Distance covered to accelerate to a given speed
4. Acceleration vs. time
5. Motor current vs. time

1. The time (t) to achieve a velocity (v):

$$\mathbf{t = [(W \cdot D \cdot v_o) / (768 \cdot n \cdot T_S)] \cdot Ln \{[1 - D \cdot F / (n \cdot 24 \cdot T_S)] / [1 - D \cdot F / (n \cdot 24 \cdot T_S) - v/v_o]\}}$$

2. The velocity (v) reached as a function of time (t):

$$\mathbf{v = v_o \cdot (1 - D\cdot F/(n\cdot 24\cdot T_S))\{1 - exp[-768\cdot n\cdot T_S\cdot t/((v_o)\cdot W\cdot D)]\}}$$

3. The distance (S) covered during acceleration to velocity (v):

$$\mathbf{S = [(W\cdot D\cdot v_o^2)/(768\cdot n\cdot T_S)]\cdot\{[(1 - D\cdot F/384\cdot T_s]\cdot Ln[(1 - D\cdot F/384\cdot T_S)/(1 - D\cdot F/384\cdot T_S - v/v_o)] - v/v_o\}}$$

4. The acceleration (a) as a function of time (t):

$$\mathbf{a = [(1 - D\cdot F/(n\cdot 24\cdot T_s))\cdot (768\cdot T_S)/(W\cdot D)]\cdot exp[-(768\cdot n\cdot T_S\cdot t)/(v_o\cdot W\cdot D)]}$$

5. The motor current (i) as a function of time (t):

$$\mathbf{i = i_o + (i_s - i_o)\{(D\cdot F/(n\cdot 24\cdot T_S))\cdot [1 - exp(-768\cdot n\cdot T_S\cdot t/v_o\cdot W\cdot D)] + exp(-768\cdot n\cdot T_S\cdot t/v_o\cdot W\cdot D)\}}$$

where,
time (**t**) is measured in seconds
weight (**W**) in lb
force (**F**) in lb; sum of external forces:
  ascending inclines: $\mathbf{F_i = W \cdot sin(\theta)}$, where $\boldsymbol{\theta}$ is the slope angle
  rolling resistance: $\mathbf{F_{rr} = W \cdot C_{rr}}$, where $\mathbf{C_{rr}}$ is the coefficient of rolling resistance
  air drag: $\mathbf{F_D = C_D\cdot A\cdot v^2/840}$, where F_D is the drag force, C_D is the drag coefficient, A is the object's frontal cross sectional area, v is the velocity
wheel diameter (**D**) in inches
velocity (**v**) in ft/sec; v_o is the no load velocity for a given wheel diameter
stall torque ($\mathbf{T_S}$) in **ft-lbs**
distance traveled (**S**) in feet
acceleration (**a**) in ft/sec^2
current (**i**) in amps; i_o is no load current; i_s is stall current
Ln() is the natural logarithm
exp() is the exponential function
n is the number of drive motors

These equations applied to the above robot with two 350W motors will tell us how long, over what distance, and the current usage during the acceleration phase, from zero to 15 mph. **Note** that the values for the no load and stall values are computed from the motor performance information and any externally applied gear ratio and *those* resultant quantities are used in the equations.

Motor Current Requirements

Looking at the motor performance graph, Figure 5, we see that for continuous operation, the motors should not draw more than about 19 amps. As the motor load changes, so will the current drawn. When a motor is first turned on, the current is limited only by the resistance and inductance of the motor armature coil and current limitations of the power supply and motor driver. As the motor spins faster, back emf increases and the motor current decreases, until the motor load is matched by the motor's torque. If the current source is unable to supply the high startup current or if it is externally limited, the motor continues to march along the motor speed vs. torque curve, perhaps slower than it would have at full current, until the available current matches the amount that corresponds to the new position on the performance curve. We expect the highest current demand during the acceleration cycle, especially uphill.

Using two motors (with a 8.6 speed reduction gear ratio); no load speed of 407 rpm; stall torque of 44 ft-lb; no load current of 1.3 amp; stall current of 112 amps we calculate the currents from Equation 5.

Level Ground

1. Time to reach 12 mph (17.6 ft/sec), t ≈ 1 sec..
   Distance covered during acceleration = 9 ft.
2. Average acceleration current over 1 second = 36 amp/motor; average power is 864W.

Uphill Sloped Ground

This is a very stressing case since in addition to climbing a steep slope, the robot is also accelerating. In this case we use the 11 mph speed the motors are capable of supplying for moving uphill. Now instead of 3.6 ft-lb of torque, the robot needs 14 ft-lb.

1. Time to reach 11 mph (16.1 ft/sec), t ≈ 1 sec. This may seem counterintuitive; why doesn't it take more time to accelerate uphill? Because the final speed is less than on level ground.
2. Distance covered during acceleration = 6 ft. The distance is less than for level ground because in the same time, the rover is going slower.
3. Average acceleration current over 1 second = 45 amp/motor; average power is 1080W.

The predicted and measured motor current profiles during the acceleration phase are shown in Figure 11. The peak measured current is limited by the 50A current sensor. The curves show the exponential decrease in current with time.

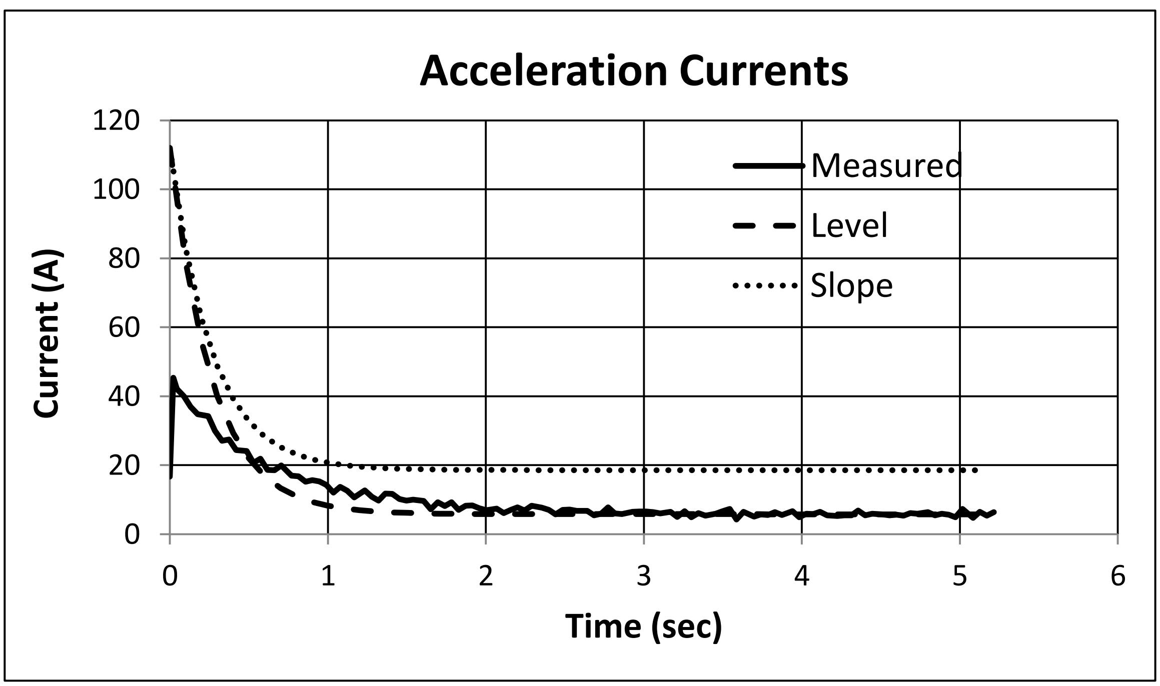

Figure 11. Acceleration Currents

With all these calculations in mind, how shall we choose the motors? We have ignored various inefficiency factors. Therefore, I always like to be conservative in my estimates and multiply my calculations by a factor of 1.5 to account for the "unknown unknowns". Another option is to settle for less demanding performance. In the present case, we estimated that at 80% efficiency we would need 310W per motor and we have selected 350 W/motor. Testing has shown that with a small change in top speed, 12 mph instead of 15 mph, the 350W motors are adequate for high speed and steep hill climbing. The acceleration current was calculated for the full, rated motor voltage. This is not the usual operating practice. If the voltage is increased more gradually from zero to the rated voltage, the acceleration time will be somewhat longer and the current demand lower.

V. <u>Battery Requirements</u>

Batteries have many characteristics including unit cell voltage, capacity, energy density, max discharge rate, depth of discharge, self-discharge rate, cycle life, thermal time constant and others that are functions of temperature and age. I can't cover all those here and battery technology continues to develop. The user should review different battery chemistries to judge the cost/benefit ratio.

We can, nevertheless, estimate the battery requirements from the current requirements determined above. The robot will need batteries that can produce high current discharge rates without over-heating or degrading the battery life, and have a large depth of discharge. The capacity of a battery is a measure of how much charge it contains when fully charged and is rated in milliamp-hours or amp-hours, which implies it is the current that the battery can supply for one hour. That is misleading. The rating is normally referred to a discharge rate over 10 (C/10 rate) or 20 (C/20 rate) hours. In other words, for a slow discharge. The more rapid the discharge, the lower the effective capacity. Not all batteries of the same chemistry with the same capacity rating deliver the same amount of power. Deep cycle batteries have a depth of discharge of 50 to 80% (can be discharged to 50 or 20% of their initial capacity) without damage or limiting the battery life. Typical lead acid automotive batteries are NOT deep cycle and after delivering starting current need to be recharged; they suffer irreversible effects when discharged to 20% of initial capacity. There are special lead acid batteries that are deep cycle; they are considerably heavier and more costly. Lithium ion batteries have a high energy density and weigh only about one third as much as lead acid batteries. They are also more costly and care must be taken in the charge and discharge rate to avoid overheating and fires. Lithium iron phosphate batteries are also deep cycle, can deliver large initial discharge rates and are safer to use than lithium ion batteries. However, their sustained discharge rate is not as great. $LiFePO_4$ batteries with a built in battery management system are available for electric bikes and should be a good compromise between lead acid and lithium ion.

The **current for the running phase,** constant speed, is taken from the equation of the motor operating curve:

$$\mathbf{i = 1.3 + 112{*}T/T_S \;=\; 1.3 + 112{*}(1 - v/v_o)}$$

For 12 mph (17.6 ft/sec), the steady state electrical current per motor is, for level ground, about 5.8 amps and for hill climbing about 19 amps, which is only slightly over the continuous allowable amount of 18 amps.

We define a run cycle as:

10 one sec accelerations:	36A x 1 sec x 10	=	100 milliAmp-Hours
running at 12 mph for 10 minutes:	7A x 10 minutes	=	1167 mA-H
20 skid steering turns :	45A x 1 sec x 20	=	250 mA-H
idling 2 minutes before the next run cycle :	1.3A x 2 minutes	=	43 mA-H

Then the total capacity needed for one run cycle, per battery would be about 1.55 A-H over 12.5 minutes (add about 12 milliamp-Hours for each *second* of steep hill climbing at constant speed). Let's assume that we want to run for 2 hours between charging. That would be about 10 run cycles, bringing the required capacity to 15.5 A-H. Multiplying the total capacity by 1.5 to account for inefficiencies or travel over undulating ground, brings the A-H rating of each battery in series to about 24 A-H. This should provide for robust operation.

VI. Summary

Two 350 W motors should meet the slightly modified performance requirement (12mph instead of 15mph). A 24v, 24 AH battery should be sufficient for several run cycles before recharge. There are a lot of estimated parameters. Testing will be required to confirm the estimates.

VII. Conclusion

You now have quantitative relationships, example calculations, and comparison measurements for a ground-based, four wheel drive robot. I hope these will help you in selecting motors for your future robots. The science of selecting robot motors is in understanding the quantitative PMDC motor performance curves and the torque equations that govern the motion. The art is in selecting appropriate performance requirements and making adjustments and accommodations in the performance criteria and the robot design.

END NOTE

Derivation of Equations of Motion

We derive the robot equations of motion during the acceleration phase by using Newton's second law of motion with the force law from the motor operating curve.

The following variables and units are used:

time (**t**) is measured in seconds. t_{TOT} is the total acceleration time.
weight (**W**) in lb
force (**F**) in lb; see below for list of external forces
wheel diameter (**D**) in inches
velocity (**v**) in ft/sec; v_o is the no load velocity for a given wheel diameter, without gearing.
stall torque ($\mathbf{T_S}$) in ft-lb
distance traveled (**S**) in feet
acceleration (**a**) in ft/sec^2 and the average acceleration ($\bar{a}$)
current (**i**) in amps; i_o is no load current; i_s is stall current

Ln() is the natural logarithm
exp() is the exponential function
n is the number of drive motors

First we derive the equations for accelerated motion with no external forces

Newton's second law of motion, $\mathbf{F = m \cdot a}$, may be written as both $\mathbf{F = m \cdot dv/dt}$ or $\mathbf{F = m \cdot v \cdot dv/dx}$.

Using the first expression the linear relationship between motor rotational speed, $\boldsymbol{\omega}$, and torque, $\mathbf{T}$, $\mathbf{T = T_S \cdot (1 - \omega/\omega_o)}$, we have:

$$\mathbf{T = F \cdot D/2 = (D/2) \cdot (W/g) \cdot dv/dt}$$

$$\mathbf{a = dv/dt = 2 \cdot T \cdot g/(W \cdot D) = 2 \cdot g \cdot T_S \cdot (1 - \omega/\omega_o)/(W \cdot D)}$$

or since $\mathbf{\omega = v/(D/2)}$, $\mathbf{dv/dt = 2 \cdot g \cdot T_S \cdot (1 - v/v_o)/(W \cdot D)}$
Straightening out the units so they are consistent and substituting 32 ft/s^2 for $\mathbf{g}$,

$$\mathbf{dv/dt = [(768 \cdot T_S)/(W \cdot D)] \cdot (1 - v/v_o)}$$

Integrating from zero to $\mathbf{v}$, and zero to $\mathbf{t}$, using the form $\int \mathbf{dx/(ax + b) = (1/a)Ln|ax + b|}$,

$$\boxed{\mathbf{v = v_o \cdot \{1 - exp[(-768 \cdot T_S \cdot t)/(v_o \cdot W \cdot D)]\}}}$$

Solving for $\mathbf{t}$,

$$\boxed{\mathbf{t = [(W \cdot D \cdot vo)/(768 \cdot Ts)] \cdot Ln[1/(1 - v/vo)]}}$$

Differentiating the expression for $\mathbf{v}$ with respect to $\mathbf{t}$,

$$\boxed{\mathbf{a = (768 \cdot T_S)/(W \cdot D) \cdot exp[(-768 \cdot T_S \cdot t)/(v_o \cdot W \cdot D)]}}$$

Using the second expression for Newton's second law, $\mathbf{F = m \cdot v \cdot dv/dx}$, we have as before:

$$\mathbf{T = Ts \cdot (1 - v/vo) = F \cdot D/2 = (D/2) \cdot (W/g) \cdot v \cdot dv/dx}$$

and

$$\mathbf{dx = W \cdot D/(2 \cdot g \cdot Ts)[v \cdot dv/(1 - v/vo)]}$$

integrating from zero to the total distance traveled, S, and from zero to v,

$$\boxed{\mathbf{S = [(W \cdot D \cdot v_o^2)/(768 \cdot T)]\{Ln[1/(1 - v/vo)] - v/vo\}}}$$

We now derive the current vs. time equation. From the motor operating curve we have:

$$\mathbf{i = i_o + (i_S - i_o) \cdot T/Ts = i_o + (i_S - i_o) \cdot (1 - v/v_o)}$$

Substituting the expression for **v** as a function of time,

$$\mathbf{i = i_o + (i_S - i_o) \cdot \{1 - [1 - \exp((-768 \cdot T_S \cdot t)/(v_o \cdot W \cdot D))]\}}$$

$$\boxed{\mathbf{i = i_o + (i_S - i_o) \cdot \exp[(-768 \cdot T_S \cdot t)/(v_o \cdot W \cdot D)]}}$$

These equations give:

1. Speed vs time
2. The time to accelerate to a given speed
3. Acceleration vs time
4. The distance covered to accelerate to a given speed
5. The motor current vs time

<u>Including external forces in the accelerated motion:</u>

Including forces due to:

1. ascending slopes: $\mathbf{F = W \cdot \sin(\theta)}$, where $\boldsymbol{\theta}$ is the slope angle
2. rolling resistance: $\mathbf{F = W \cdot C_{rr}}$, where $\mathbf{C_{rr}}$ is the coefficient of rolling resistance
3. air drag: $\mathbf{F_D = C_D \cdot A \cdot v^2/840}$, where F_D is the drag force, C_D is the drag coefficient, A is the object effective cross sectional area, v is the velocity

adds additional terms in the equations of motion.

The original equation of motion: $\mathbf{T = (D/2) \cdot (W/g) \cdot dv/dt}$**,** is modified by adding the torques due to the above forces to the right side of the equation. This represents the forces in addition to the acceleration of mass that the motor torque, **T**, must work against. We represent these forces as a composite term, **F**. Now we must solve the equation:

$$\mathbf{T = (D/2) \cdot (W/g) \cdot dv/dt + F}$$

We proceed as before, using a different indefinite integral, and derive:

Time (t) versus Velocity (v):

$$\boxed{\mathbf{t = [(W \cdot D \cdot v_o)/(768 \cdot n \cdot T_S)] \cdot Ln\,\{[1 - D \cdot F/(n \cdot 24 \cdot T_S)] / [1 - D \cdot F/(n \cdot 24 \cdot T_S) - v/v_o]\}}}$$

Velocity (v) vs. Time (t):

$$\boxed{\mathbf{v = v_o \cdot (1 - D \cdot F/(n \cdot 24 \cdot T_S))\,\{1 - \exp[-768 \cdot n \cdot T_S \cdot t/((v_o) \cdot W \cdot D)]\}}}$$

Distance (S) vs. velocity (v):

$$\mathbf{S = [(W\cdot D\cdot v_o^2)/(768\cdot n\cdot T_S)]\cdot\{[(1 - D\cdot F/384\cdot T_s]\cdot Ln[(1 - D\cdot F/384\cdot T_S)/(1 - D\cdot F/384\cdot T_S - v/v_o)] - v/v_o\}}$$

Acceleration (a) vs. Time (t):

$$\mathbf{a = [(1 - D\cdot F/(n\cdot 24\cdot T_s)) \cdot (768\cdot T_S)/(W\cdot D)]\cdot exp[-(768\cdot n\cdot T_S\cdot t)/(v_o\cdot W\cdot D)]}$$

Current (i) vs. Time (t):

$$\mathbf{i = i_o + (i_s - i_o)\ \{(D\cdot F/(n\cdot 24\cdot T_S)) \cdot [1 - exp(-768\cdot n\cdot T_S\cdot t/v_o\cdot W\cdot D)] + exp(-768\cdot n\cdot T_S\cdot t/v_o\cdot W\cdot D)\}}$$