The Denavit Hartenberg Convention

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1 Why do Denavit Hartenberg (DH)?

Last class, Matt did forward kinematics for the simple RR arm. The process seemed intuitive and easy. But the drawback with such a technique is that it is not amenable to automation. We would like to have a procedure that is handy for complex mechanisms also.

2 A Systematic Approach to Forward Kinematics

Ideally, we would like to know the position and orientation of each link as the mechanism moves. This can be done by attaching frames to each link and computing how the frames change as the robot moves. For this, we need to know how the frames change with the mechanism parameters. Intuitively, it seems good to parametrize the relationship between two adjacent links (frames), so that we can move from one link to another. Thus, ultimately, we will have a table of parameters that defines the relationship between two adjacent frames (links).

Here are the steps:

1. Attach Frame i to Link i.

2. Define ${}^{(i-1)}_{i}T(q_i)$ for each link, where q_i is the joint coordinate θ_i or d_i for revolute and prismatic joints respectively.

3. Compute ${}^{0}_{N}T = {}^{0}_{1}T \ \cdots \ {}^{(N-1)}_{N}T$

The frame assignment can be done any way, but it makes sense to have a systematic procedure for fixing frames onto links.

2.1 The Denavit Hartenberg Rules

There are two versions of the DH rules.

1. Proximal

2. Distal

The main difference between the two is the labelling of the indices. We will be using the Proximal system.

Here are the DH frame-fixing rules (from Craig p77):

1. Identify the Joint axes and imagine infinite lines along them. For steps 2-5, consider two of these neighboring lines (at axes i and $(i+1)$).

2. Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the ith axis, assign the frame origin.

3. Assign axis $\hat{Z}_i$ pointing along the joint axis i .

4. Assign axis $\hat{X}_i$ pointing along the common perpendicular, or if the axes intersect, assign $\hat{X}_i$ to be normal to the plane containing the two axes.

5. Assign $\hat{Y}_i$ to complete a right-hand coordinate system.

6. Assign $\{0\}$ to match $\{1\}$ when the first joint variable is zero. For N, choose an origin location and $\hat{X}_N$ direction freely, keeping as many linkage parameters zero.

Aside: Lines in Space

The relationship between two lines in space is given by the common perpendicular between them (Figure 1). a gives the shortest distance between the lines and α the angle between the lines.
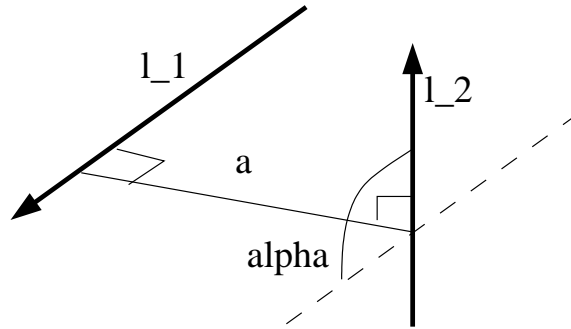

Figure 1: Common Perpendicular of Two Lines

2.2   Formula for $_i^{(i-1)}T$

It is noticed that frame $_iT$ can be reached from $_{(i-1)}T$ by executing the following motions all in the moving frame (Figure 2).

Step #1: Rotate about $X_{(i-1)}$ by $\alpha_{(i-1)}$.

Step #2: Translate along $X_{(i-1)}$ by $a_{(i-1)}$.

Step #3: Rotate about Z_i by θ_i.

Step #4: Translate along Z_i by d_i.

Thus, we have
$$_i^{(i-1)}T(\alpha,a,\theta,d) = \mathbf{Rot}(\alpha_i, X_{(i-1)}) \cdot \mathbf{Trans}(a_{(i-1)}, X_{(i-1)}) \cdot \mathbf{Rot}(\theta_i, Z_i) \cdot \mathbf{Trans}(d_i, Z_i)$$

Now that we have a expression for $_i^{(i-1)}T$, we can easily compute $_i^{(i-1)}T$ (the 4x4 matrix) by multiplying the matrices out to get eqn. 3.6 (Craig p84).
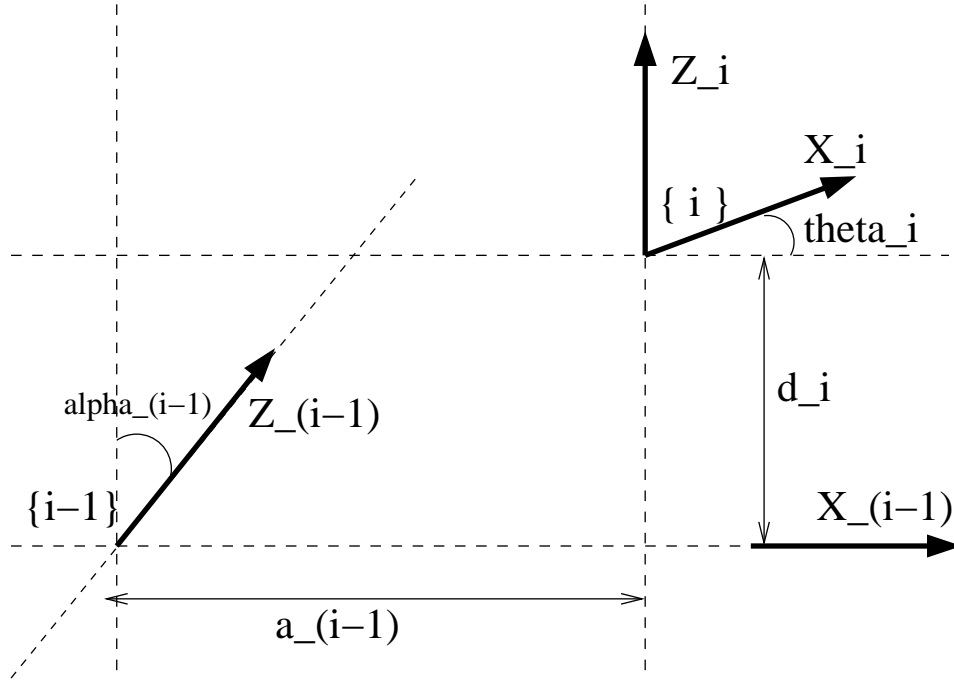
Figure 2: Transform from Frame i-1 to i

2.3 Physical Meaning of DH parameters

If the DH frames are assigned properly, then :

- a_i = distance from $\hat{Z}_i$ to $\hat{Z_{i+1}}$ measured along $\hat{X}_i$

- α_i = angle between $\hat{Z}_i$ and $\hat{Z_{i+1}}$ measured about $\hat{X}_i$

- d_i = distance from $\hat{X_{i-1}}$ to $\hat{X}_i$ measured along $\hat{Z}_i$

- θ_i = angle between $\hat{X_{i-1}}$ and $\hat{X}_i$ measured about $\hat{Z}_i$

3 Example: Two Link RR Arm

Now, let's do an example. Lets compare the answers from two techniques for the Revolute-Revolute arm.

3.1 Plain Old Trigonometry

Referring to Figure 3, we have

$$
\begin{aligned}
x &= l_1 c_1 + l_2 c_{12} \\
y &= l_1 s_1 + l_2 s_{12}
\end{aligned}
\tag{1}
$$

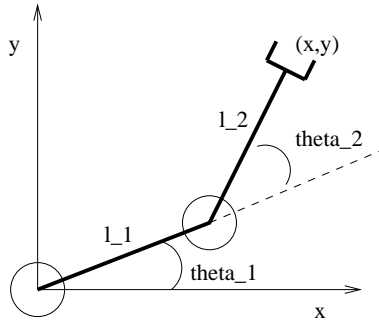Figure 3: RR Arm

3.2 Denavit Hartenberg

We follow the rules outlined by the convention. Refer to Figure 4. We note the following:

1. $\hat{Z}_1, \hat{Z}_2$ on joint axes 1 and 2 respectively.

2. $\hat{Z}_0 = \hat{Z}_1$

3. $\hat{X}_0$ common perpendicular to $\hat{Z}_0$ and $\hat{Z}_1$.

4. $\hat{X}_1$ common perpendicular to $\hat{Z}_1$ and $\hat{Z}_2$.

5. $\hat{X}_2$ intersects $\hat{X}_1$.

Here are the DH parameters for this assignment.

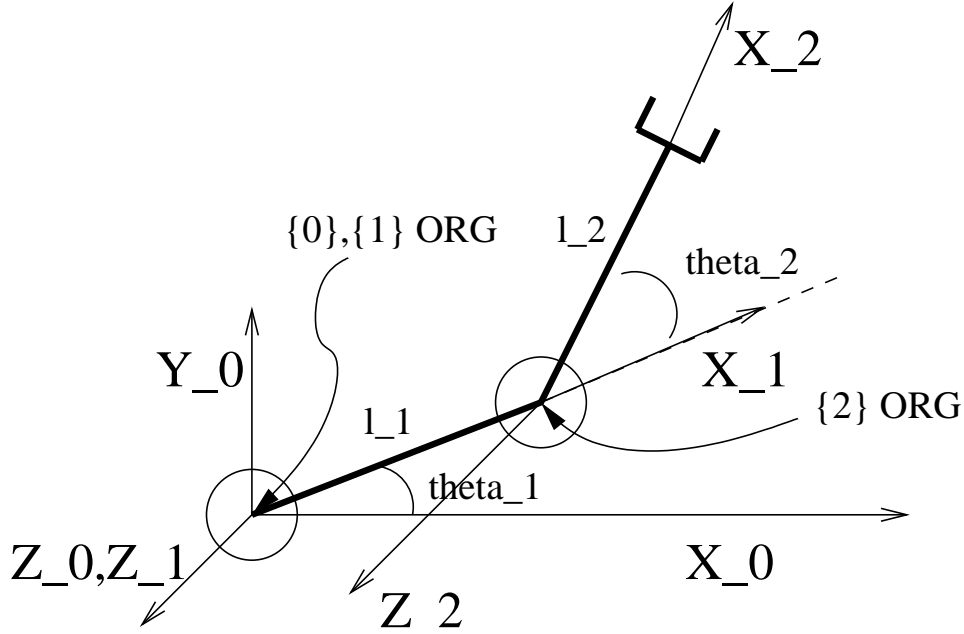| i	α	a	θ	d
${}^1_0T$	0	0	θ_1	0
${}^2_1T$	0	l_1	θ_2	0

Figure 4: RR Arm with DH Frames Attached

Now, filling in the elements of the formula, we get

$$
\begin{aligned}
{}^{0}_{2}T &= {}^{0}_{1}T \cdot {}^{1}_{2}T \\[2mm]
&= \begin{pmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} c_2 & -s_2 & 0 & l_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\[2mm]
&= \begin{pmatrix} c_1 c_2 - s_1 s_2 & -c_1 s_2 - s_1 c_2 & 0 & l_1 c_1 \\ s_1 c_2 + c_1 s_2 & -s_1 s_2 + c_1 c_2 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\[2mm]
&= \begin{pmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 \\ s_{12} & c_{12} & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}
$$

The fourth column of ${}^{0}_{2}T$ gives the coordinates of ${}^{0}_{2}P_{ORG}$ and is not the same answer as in the plain trigonometry case; this is because we are considering two different points. This is easily remedied by adding an extra *tool* frame, {T} (Figure 5) defined as:

$$
{}^{2}_{T}T = \begin{pmatrix} 0 & 0 & 1 & l_2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
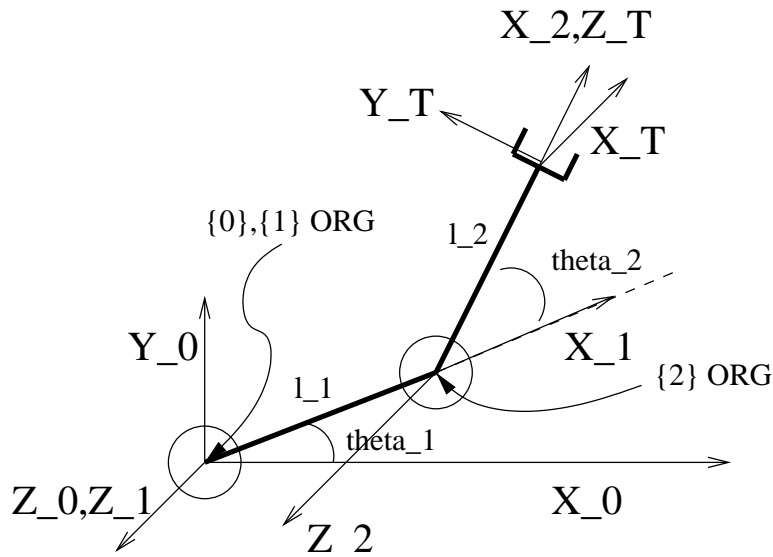$$

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Figure 5: RR Arm with DH and Tool Frames Attached

Thus, we get

$$
\begin{aligned}
{}^0_T T &= {}^0_2 T \cdot {}^2_T T \\
&= \begin{pmatrix}
0 & -s_{12} & c_{12} & l_1 c_1 + l_2 c_{12} \\
0 & c_{12} & s_{12} & l_1 s_1 + l_2 s_{12} \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\end{aligned}
\tag{2}
$$

Note that the answers match between eqns. (1) and (2).

4 When is DH useful?

Clearly, for a simple mechanism such as the RR arm, using DH is a big overhead as compared to simple trigonometry. But for complex mechanisms, DH is far easier to automate. Also, once the frames are fixed and the parameters identified, it is easy to find the relationship between any two frames on the mechanism. Also, the DH parameters are an easy way of completely specifying the mechanism. We will see more of DH in future classes.